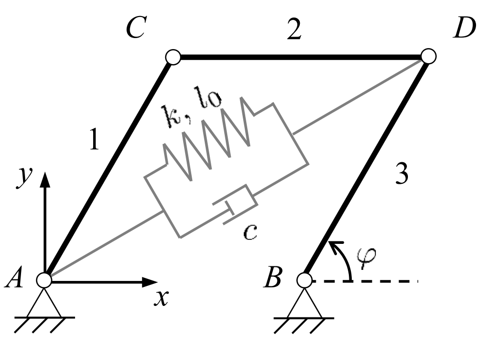
LINEARIZATION OF THE DYNAMICS OF A FOUR-BAR LINKAGE WITH A SPRING AND A DAMPER ON ITS DIAGONAL

Figure 1 shows a four-bar linkage with a linear spring and a damper that connect points *A* and *D* on its diagonal. The system is subjected to gravity effects (9.81 m/s^2 along the negative direction of the *y*-axis). The three rods, *A*-*C*, *C*-*D*, and *B*-*D* have identical physical properties: mass m_f and length l.



Physical properties

Spring

$k = 25$ N/m

$l_0 = \sqrt{2}$ m

Damper

$c = 1$ Ns/m

Rod length, l: 1 m

Rod mass, m_f: 1 kg

Figure 1: A four-bar linkage with a spring and a damper

The dynamics of this system can be described by means of a single ODE in terms of the angle φ from rod *B*-*D* to the global *x*-axis:

$$\frac{5}{3}m_f l^2\ddot{\varphi} + 2m_f g l\cos\varphi - kl\sin\varphi\left(l - \frac{l_0}{\sqrt{2(1+\cos\varphi)}}\right) + \frac{cl^2\dot{\varphi}\sin\varphi}{\sqrt{2(1+\cos\varphi)}}\sin\frac{\varphi}{2} = 0$$

The system is in static equilibrium ($\dot{\varphi} = 0, \ddot{\varphi} = 0$) at an angle φ_0. The linearized dynamics about this equilibrium configuration is given by

$$m_e\delta\ddot{\varphi} + c_e\delta\dot{\varphi} + k_e\delta\varphi = 0$$

where

$$m_e = \frac{5}{3}m_f l^2$$

$$c_e = \frac{cl^2\sin\varphi_0}{\sqrt{2(1+\cos\varphi_0)}}\sin\frac{\varphi_0}{2}$$

$$k_e = -2m_f g l\sin\varphi_0 - kl^2\cos\varphi_0 + \frac{kll_0}{\sqrt{2(1+\cos\varphi_0)}}\left(\cos\varphi_0 + \frac{\sin^2\varphi_0}{2(1+\cos\varphi_0)}\right)$$

This linearized dynamics equation has a pair of complex conjugate eigenvalues $s_{1,2} = d \pm i\cdot\omega_d$. These represent the exact spectrum of the system.

Multibody dynamics formulations can express the system dynamics either as a set of ODEs or a system of DAEs. Depending on the coordinate selection and the formulation properties, the linearized dynamics can be a linear system with the exact spectrum of the constrained system, or with an approximation of it. Spurious eigenvalues may also be obtained; these must be discriminated from the real system spectrum.

For benchmarking purposes, the accuracy of a linearization method is defined as the norm-2 of the 2×1 array that contains the difference between the exact eigenvalues $s_{1,2}$ and the true eigenvalues predicted by the selected method. Given the small system size, the efficiency of the method is defined as the time elapsed in 1000 executions of the linearization process, which includes the evaluation of the necessary linearization terms, the evaluation of the system eigenvalues, and the discrimination between true and spurious eigenvalues, if necessary. The solution file should include the eigenvalues yielded by the method, both the real and the spurious ones.

Reference numerical values

For an equilibrium angle $\varphi_0 = 2.23433101898$ rad, the value of the system eigenvalues evaluated with the analytical expression is $s_{1,2} = -0.2423858093 \pm 2.1339550282 \cdot i$.

References

[1] F. González, P. Masarati, and J. Cuadrado. On the linearization of multibody dynamics formulations. *Proceedings of the ASME 2016 IDETC/CIE*, paper 59277, Charlotte, NC, USA, 2016.